Search with Interdependent Values

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Motivation: Search Theory

Search with Independent Value

- How search frictions affect market outcomes
  - Transaction frictions
  - Informational frictions: imperfect information (prices, match quality, etc.)
    $\Rightarrow$ can be discovered at a search cost
- A key assumption for traditional search theory: independence

Motivation: Pandora's Box (Weitzman, 1979)

Search with Independent Value

- N Pandora's boxes indexed by $i = 1, 2, ..., N$
  - Payoff: box i's value $V_i \sim F_i(\cdot)$, *independent* of other boxes
  - Search: realization v_i can be observed only if box i is opened, incurring *search cost* s
- The agent sequentially opens the boxes, with questions on...
  1. Whether to open another box? $\Rightarrow$ Stopping rule
  2. In which order to open the remaining closed boxes? $\Rightarrow$ Search order

Motivation: Pandora's Box (Weitzman, 1979)

Search with Independent Value

- Reservation price z_i for box i: the level of current utility that makes her indifferent between whether to open it:

$$s = \int_{z_i}^{+\infty} (\varepsilon - z_i)\, dF_i(\varepsilon)$$

  — *solely depends on box i's characteristics*

- Equilibrium
  - Optimal stopping rule - terminate search iff the maximum sampled reward exceeds the reservation price of every closed box $\Rightarrow$ *an upper threshold captures "luck" effect*
  - Optimal search order - open boxes in the descending order of z_i $\Rightarrow$ *max. luck effect*

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- Equilibrium
  - Optimal stopping rule - terminate search iff the maximum sampled reward exceeds the reservation price of every closed box ⇒ *an upper threshold captures "luck" effect*
  - Optimal search order - open boxes in the descending order of z_i ⇒ *max. luck effect*
  - ▷ Corollary 1. No return (until open all boxes) not seen in experiment (Kogut, 1990)
  - ▷ Corollary 2. History-independent search order

Motivation: Interdependent Values Search

- What if the values of boxes are correlated? For example,
  - Product market: cars using the same engine; computers using the same CPU
  - Labor market: students from the same college
  - Financial market: assets suffer from the common, aggregate shock
- We are examining the following case

$$V_i = X + E_i$$

  - Common attribute $X \sim F_X(\cdot)$
  - Idiosyncratic attribute $E_i \sim F_{E_i}(\cdot)$

Main Results

Search with Interdependent Value

- Two mechanisms
  - Luck effect - same as the literature
  - Cockroach effect - a poor realization provides information on common attribute

    "When we see one cockroach, there are likely many more that we have not seen"
- Optimal stopping rule
  - Luck effect ⇒ too good to continue
  - Cockroach effect ⇒ too bad to continue - *signal of a bad common attribute*
- Optimal search order
  - Luck effect ⇒ prioritizing larger-variance box
  - Cockroach effect ⇒ prioritizing small-variance box - *contains better information*

Main Results

Comparison to Pandora's Rule

	Independent values	Interdependent values
Stopping rule	terminate when $v_i > z_i$	when $v_i > \overline{v}_i$ or $v_i < \underline{v}_i$
Search order	prioritize larger-σ box	small-σ box when uncertain
Return	no return	return w/o opening all boxes
History	history-independent	history-dependent

Model Setup

Setup

- Two boxes: $V_i = X + E_i,\ i \in \{1, 2\}$
- The densities $\{f_X(\cdot|V_i = v)\}$ and $\{f_{E_i}(\cdot|V_i = v)\}$ have the strict monotone likelihood ratio property (MLRP)

  $\Rightarrow$ *a better realization implies better belief for both common and idiosyncratic attributes*

- Sequential search, search cost s, with first sampling free
- After opening box i, only the gross value v_i is observable

Timing

1. The consumer chooses box $a \in \{1, 2\}$ to sample (first free) $\Rightarrow$ search order

$$U = \max_{a \in \{1,2\}} \left\{ \mathbb{E}_{v_a} \left[U_1 \left(v_a; a \right) \right] \right\}$$

2. Observing v_a, she decides whether to stop $\Rightarrow$ subgame: stopping rule

$$U_1(v_a; a) = \max \left\{ \underbrace{\max\{0, v_a\}}_{\text{Stop}}, \underbrace{-s + \mathbb{E}_{V_b} \left[U_2 \left(v_a, V_b \right) | V_a = v_a \right]}_{\text{Continue}} \right\}$$

3. If she samples box b in stage 2, she will choose the best option

$$U_2(v_a, v_b) = \max \left\{ 0, v_a, v_b \right\}$$

Optimal Stopping Rule

Optimal Stopping Rule

General Description

- Given the observed value of box a as v_a
- Keep searching iff incremental benefit exceeds additional search cost:

$$g_a(v_a) := \underbrace{\mathbb{E}_{V_b}\Big[\max\{0, v_a, V_b\}\Big| V_a = v_a\Big]}_{\text{(A) Information}} - \underbrace{\max\{0, v_a\}}_{\text{(B) Outside option}} > s$$

- Two channels of v_a
  - (A) Information: influence belief on X and E_a, and hence V_b
  - (B) Outside option: when $v_a > 0$, better v_a means better outside option

Optimal Stopping Rule

Lower Threshold - when $v_a < 0$

- When $v_a < 0$, box a has no influence over outside option $\Rightarrow \max\{0, v_a\} = 0$
- Equivalent condition for searching:

$$g_a\left(v_a\right) = \mathbb{E}_{V_b}\left[\max\{0, \underbrace{X + E_b}_{V_b}\} \middle| V_a = v_a\right] > s \quad , \quad \forall\, v_a \leq 0$$

- MLRP $\quad \Rightarrow \quad g_a(-\infty) = 0 \quad \text{and} \quad g_a'(v_a) > 0,\ \forall v_a < 0$

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$\therefore$ If $s < g_a(0)$, $\exists$ a unique $\underline{v}_a < 0$ such that she will stop sampling iff $v_a < \underline{v}_a$

Optimal Stopping Rule

Upper Threshold - when $v_a > 0$

- When $v_a > 0$, both information and outside option channels function
- Equivalent condition for searching:

$$g_a(v_a) = \mathbb{E}_{E_b}\left[\max\{0, \underbrace{E_b - E_a}_{V_b - V_a}\}\Big| V_a = v_a\right] > s \quad , \quad \forall\, v_a \geq 0$$

- MLRP $\quad \Rightarrow \quad g_a(+\infty) = 0 \quad$ and $\quad g_a'(v_a) < 0, \ \forall v_a > 0$

Optimal Stopping Rule

Upper Threshold - when $v_a > 0$

- When $v_a > 0$, both information and outside option channels function
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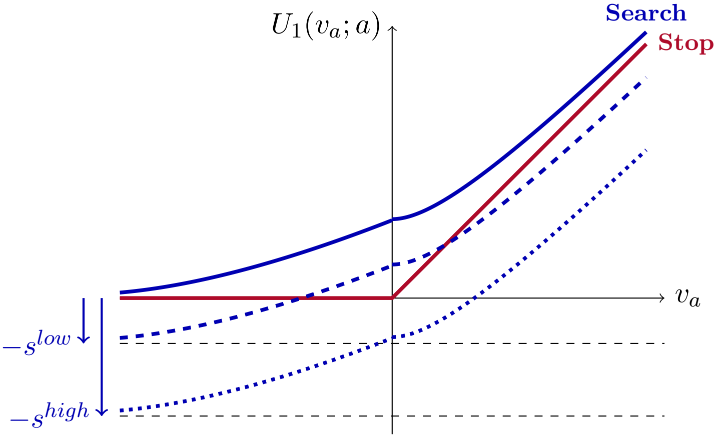
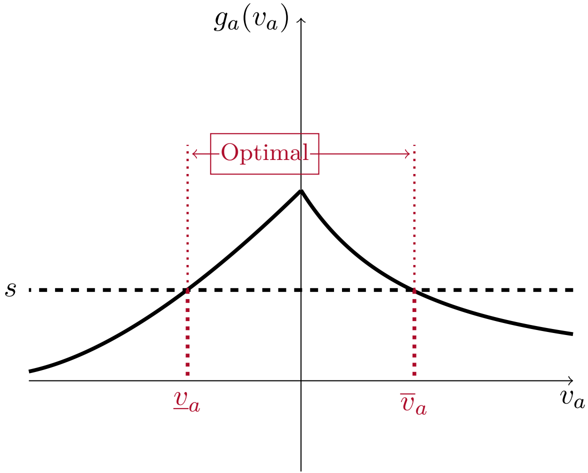
Optimal Stopping Rule



Information Value



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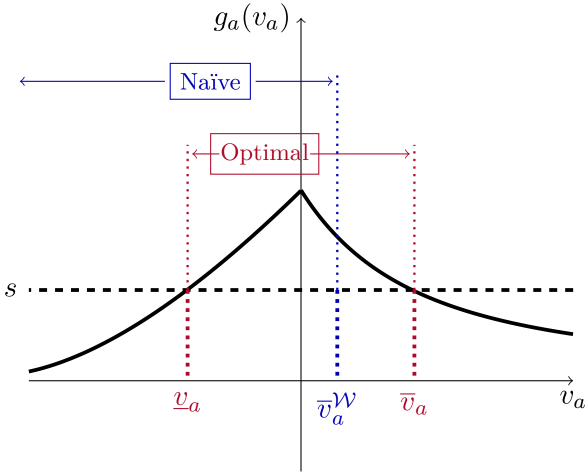
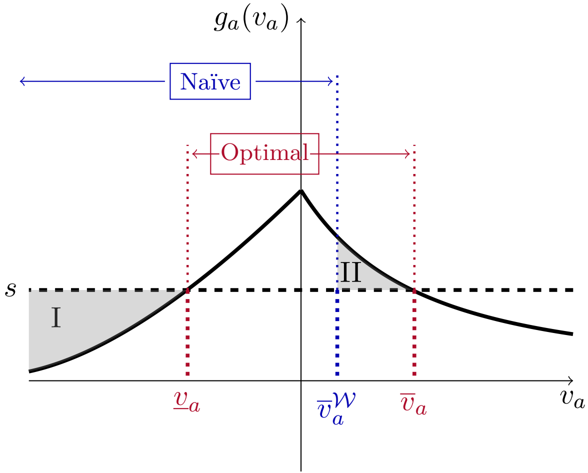


Information Value



Optimal Search Order

Expected Utility

- Take the first best case with $s = 0$ as benchmark, we have:

$$\mathbb{E}_{v_a}[U_1(v_a;a)] = U^{fb} - \int_0^s \Big[F_{V_a}(\overline{v}_a(t)) - F_{V_a}(\underline{v}_a(t))\Big]\,dt$$

⇒ Difference in expected utility between first sampling box 2 and 1:

$$\Delta U = \underbrace{\int_0^s \Big[F_{V_1}(\overline{v}_1(t)) - F_{V_2}(\overline{v}_2(t))\Big]\,dt}_{\Delta \text{ Luck effect}} + \underbrace{\int_0^s \Big[F_{V_2}(\underline{v}_2(t)) - F_{V_1}(\underline{v}_1(t))\Big]\,dt}_{\Delta \text{ Cockroach effect}}$$

Optimal Search Order

When Variances Differ - $\sigma_1 < \sigma_2$

- Assume $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $E_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$
- Differ in idiosyncratic variances: $\mu_i = 0$ and $\sigma_1 < \sigma_2$
- Trade-off between the two effects: when $\mu_X > \sqrt{\frac{\sigma_X^2\left(\sigma_1^2+\sigma_2^2\right)+\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_X^2}}$,
  - Luck effect: the larger-variance box 2 dominates $\Leftarrow F_{V_1}(\overline{v}_1(t)) > F_{V_2}(\overline{v}_2(t))$
  - Cockroach effect: the smaller-variance box 1 dominates $\Leftarrow F_{V_1}(\underline{v}_1(t)) > F_{V_2}(\underline{v}_2(t))$

Optimal Search Order

When Variances Differ - $\sigma_1 < \sigma_2$

- Depending on the variance of common attribute σ_X...
  - when σ_X is small enough, sampling box 2 first is optimal
    $\Rightarrow$ already has good ex-ante information on X
  - when σ_X is large enough, sampling box 1 first is optimal
    $\Rightarrow$ need better information on X from searching

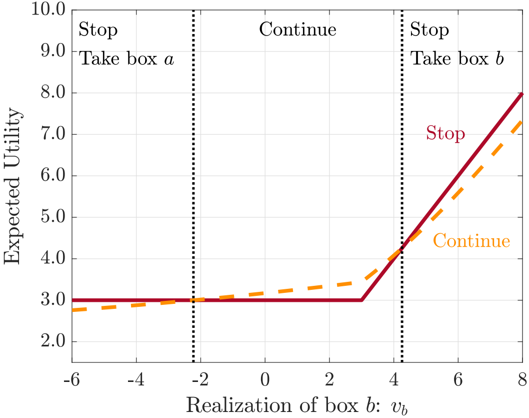
Extension

Simulation in Three-Box Model

Return w/o Sampling All Goods

An Example with Three Boxes

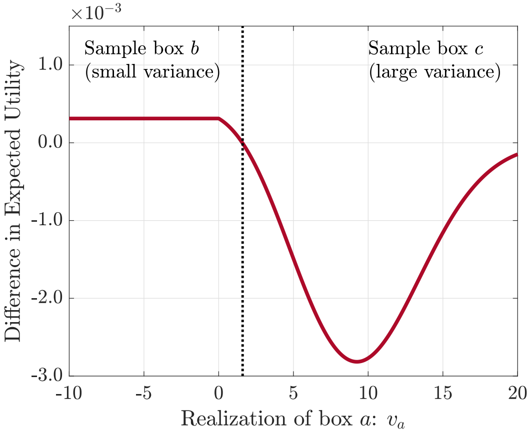
- After sampling box a and seeing $v_a = 3$...



History-Dependent Search Order

An Example with Three Boxes

- When σ_X is large, $\sigma_a \to +\infty$ and $\sigma_b < \sigma_c$...



Conclusion

Takeaways

Search with interdependent values

- Luck effect and cockroach effect
  - $\Rightarrow$ Two cutoff values in the optimal stopping rule
  - $\Rightarrow$ Trade-off in choosing search order
  - $\Rightarrow$ Return without sampling all boxes
  - $\Rightarrow$ History-dependent search order

Future Work

- Generalize the model: issues to overcome
  - N boxes: correlated bandits problem — hard to solve analytically
  - using general distribution in search order — hard to compare expected utility
- Empirical evidence

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